4.1 Self-inductance

If the current through a coil is altered then the flux through that coil also changes, and this will induce an e.m.f. in the coil itself. This effect is known self-induction and the property of the coil is the self-inductance \( L \) of the coil, usually abbreviated as the inductance. A current-carrying coil produces a magnetic field that links its own turns. If the current in the coil changes the amount of magnetic flux linking the coil changes and, by Faraday's law, an emf is produced in the coil. This emf is called a self-induced emf. Let the coil have \( N \) turns. Assume that the same amount of magnetic flux \( \Phi \) links each turn of the coil. The net flux linking the coil is then \( N \Phi \). This net flux is proportional to the magnetic field, which, in turn, is proportional to the current \( I \) in the coil. Thus we can write \( N \Phi \propto I \). This proportionality can be turned into an equation by introducing a constant. Call this constant \( L \), the self-inductance (or simply inductance) of the coil

\[
N \Phi = LI \quad \text{or} \quad L = \frac{N \Phi}{I}
\]

As with mutual inductance, the unit of self-inductance is the henry.

The self-induced emf can now be calculated using Faraday's law

\[
\mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} = -\frac{\Delta (N \Phi)}{\Delta t} = -N \frac{\Delta (LI)}{\Delta t} = -L \frac{\Delta I}{\Delta t}
\]

The above formula is the emf due to self-induction.

Formula for the self-inductance of a solenoid of \( N \) turns, length \( l \), and cross-sectional area \( A \). Assume that the solenoid carries a current \( I \). Then the magnetic flux in the solenoid is

\[
\Phi = \mu_0 \frac{NI}{l} A. \quad L = \frac{N \Phi}{I} = \frac{N}{l} \mu_0 \frac{NI}{l} A
\]

\[
L = \mu_0 \frac{N^2}{l} A \quad \text{or} \quad [L = \mu_0 n^2 A l] \quad \text{where } n = \frac{N}{l}.
\]

(Note how \( L \) is independent of the current \( I \).)

The energy is used to produce the magnetic field in and around the coil. If the current is suddenly interrupted a spark may occur as the energy is dissipated. Self-inductance can be a problem in circuits, where the breaking of the circuit can induce a large e.m.f., and so the switches maybe immersed in oil to quench the arc. Alternatively a capacitor may be connected across the terminals to slow down the decay of current and so reduce the induced e.m.f.
4.2 Growth and decay of current in an inductor

When a battery of e.m.f $E$ is connected across a resistor and an inductor in series the current does not rise to its final value instantaneously. There is a rise time that is due to the back e.m.f in the inductor and the resistance and inductance of the circuit.

$$I = \frac{E}{R}(1 - e^{-t/(L/R)})$$

A similar argument can be applied to the decay of current when the cell is disconnected, the equation in this case being -

$$L \frac{dI}{dt} = IR$$

with solution:

$$I = \frac{E}{R}(e^{-t/(L/R)})$$

Example

A coil with a self-inductance of 1.25 mH (see previous examples) is connected to a resistor of 24 $\Omega$ and a supply of 12 V. Calculate the time taken for the current to rise to 0.45A. (Final d.c current would be 0.5A in this circuit).

Using the equation $I = \frac{E}{R}(1 - e^{-t/(L/R)})$  
Therefore $0.1 = e^{-t/(0.00125/24)}$ and so  
$10 = e^{t/(0.00125/24)}$  
and therefore  
$t = (2.3x0.001250/24 = 1.1 \times 10^{-4}s$

Example

Calculate the energy stored in the coil given in the previous example ($L = 1.25$ mH) if the current through it is 0.5 A.

Energy stored = $\frac{1}{2} LI^2 = 1.25\times10^{-3}\times0.5^2 = 0.31\times10^{-3}$ J
4.3 Mutual Inductance

Suppose we hook up an AC generator to a solenoid so that the wire in the solenoid carries AC. Call this solenoid the primary coil. Next place a second solenoid connected to an AC voltmeter near the primary coil so that it is coaxial with the primary coil.

The alternating current in the primary coil produces an alternating magnetic field whose lines of flux link the secondary coil (like thread passing through the eye of a needle). Hence the secondary coil encloses a changing magnetic field. By Faraday's law of induction this changing magnetic flux induces an emf in the secondary coil. This effect in which changing current in one circuit induce an emf in another circuit is called mutual induction Fig 4.2 mutual inductance

When the current in a coil is changing an e.m.f will be induced in a nearby circuit due to some of the magnetic flux produced by the first circuit linking the second. The phenomenon is known as mutual induction. It is important to realise that the induced e.m.f. lasts only as long as the current in the first circuit is changing as shown in Fig 4.2

The mutual inductance M is defined by the equation

\[ M = \frac{\mu_0 N_1 N_2}{x} \]

where \( E \) is the e.m.f induced in the secondary coil and \( \frac{dI}{dt} \) the rate of change of current in the primary.

Consider the mutual inductance of a long solenoid and a coil as shown in the diagram. Suppose that a short coil of \( N_2 \) turns is wound round a solenoid of \( N_1 \) turns, with a cross-sectional area \( A \), length \( x \) and carrying a current \( I \) The flux at the centre of the solenoid is:

\[ \text{Mutual inductance } (M) = \frac{\mu_0 AN_1 N_2}{x} \]

Example

Calculate the mutual inductance of a pair of coils if the primary has 1000 turns of radius 2 cm and is 1 m long while the secondary has 1200 turns and is wound round the centre of the primary.

\[ M = 4\pi \times 10^{-7} \times 4 \times 10^{-4} \times 1000 \times 1200 \]

\[ = 1.90 \times 10^{-3} \text{ H} = 1.90 \text{ mH} \]
Let the primary coil have \(N_1\) turns and the secondary coil have \(N_2\) turns. Assume that the same amount of magnetic flux \(\Phi_2\) from the primary coil links each turn of the secondary coil. The net flux linking the secondary coil is then \(N_2\Phi_2\). This net flux is proportional to the magnetic field, which, in turn, is proportional to the current \(I\) in the primary coil. Thus we can write \(N_2\Phi_2 \propto I\). This proportionality can be turned into an equation by introducing a constant. Call this constant \(M\), the mutual inductance of the two coils:

\[
N_2\Phi_2 = MI_1 \quad \text{or} \quad M = \frac{N_2\Phi_2}{I_1}
\]

The unit of inductance is \(\frac{\text{wb}}{\text{A}} = \text{henry (H)}\) named after Joseph Henry.

The emf induced in the secondary coil may now be calculated using Faraday’s law:

\[
\varepsilon_2 = -N_2 \frac{\Delta \Phi_2}{\Delta t} = - \frac{\Delta (N_2\Phi_2)}{\Delta t} = - \frac{\Delta (MI_1)}{\Delta t} = -M \frac{\Delta I_1}{\Delta t}
\]

The above formula is the emf due to mutual induction.

4.4 Dot Rule:

![Fig4.4 Dot Rule](image)
Problem 1:

For the circuit shown in Fig. 13.3, (a) determine \( v_1 \) if \( i_2 = 5 \sin 45t \) A and \( i_1 = 0 \); (b) determine \( v_2 \) if \( i_1 = -8e^{-t} \) A and \( i_2 = 0 \).

The dot convention provides a relationship between the terminal at which a current enters one coil, and the positive voltage reference for the other coil.
4.5 Inductors in Series

Inductors can be connected together in either a series connection, a parallel connection or combinations of both series and parallel together, to produce more complex networks whose overall inductance is a combination of the individual inductors. However, there are certain rules for connecting inductors in series or parallel and these are based on the fact that no mutual inductance or magnetic coupling exists between the individual inductors as shown in Fig 4.5

![Fig4.5 Inductors in Series](image)

The current, \( I \), that flows through the first inductor, \( L_1 \), has no other way to go but pass through the second inductor and the third and so on. Then, inductors in series have a Common Current flowing through them, for example:

\[
l_{L1} = l_{L2} = l_{L3} = l_{AB} \ldots \text{etc.}
\]

In the example above, the inductors \( L_1, L_2 \) and \( L_3 \) are all connected together in series between points A and B. The sum of the individual voltage drops across each inductor can be found using Kirchoff’s Voltage Law (KVL) where, \( V_T = V_1 + V_2 + V_3 \) and we know from the previous tutorials on inductance that the self-induced emf across an inductor is given as: \( V = L \frac{di}{dt} \).

So by taking the values of the individual voltage drops across each inductor in our example above, the total inductance for the series combination is given as:
By dividing through the above equation by \( \frac{di}{dt} \) we can reduce it to give a final expression for calculating the total inductance of a circuit when connecting inductors in series and this is given as:

\[
L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}
\]

4.5.1 Inductors in Series Equation

Then the total inductance of the series chain can be found by simply adding together the individual inductances of the inductors in series just like adding together resistors in series. However, the above equation only holds true when there is "NO" mutual inductance or magnetic coupling between two or more of the inductors, (they are magnetically isolated from each other).

One important point to remember about inductors in series circuits, the total inductance \( L_T \) of any two or more inductors connected together in series will always be greater than the value of the largest inductor in the series chain.

4.5.2 Mutually Connected Inductors in Series

When inductors are connected together in series so that the magnetic field of one links with the other, the effect of mutual inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other. Mutually connected inductors in series can be classed as either “Aiding” or “Opposing” the total inductance. If the magnetic flux produced by the current flows through the coils in the same direction then the coils are said to be Cumulatively Coupled. If the current flows through the coils in opposite directions then the coils are said to be Differentially Coupled as shown below.

4.5.3 Cumulatively Coupled Series Inductors

While the current flowing between points A and D through the two cumulatively coupled coils is in the same direction, the equation above for the voltage drops across each of the coils needs to be modified to take into account the interaction between the two coils due to the effect of mutual inductance. The self inductance of each individual coil, \( L_1 \) and \( L_2 \) respectively will be the same as before but with the addition of \( M \) denoting the mutual inductance.
Then the total emf induced into the cumulatively coupled coils is given as:

\[
L_T \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2\left(M \frac{di}{dt}\right)
\]

Where: \(2M\) represents the influence of coil \(L_1\) on \(L_2\) and likewise coil \(L_2\) on \(L_1\).

By dividing through the above equation by \(di/dt\) we can reduce it to give a final expression for calculating the total inductance of a circuit when the inductors are cumulatively connected and this is given as:

\[
L_{\text{total}} = L_1 + L_2 + 2M
\]

If one of the coils is reversed so that the same current flows through each coil but in opposite directions, the mutual inductance, \(M\) that exists between the two coils will have a cancelling effect on each coil as shown below.

**4.5.4 Differentially Coupled Series Inductors**

![Differentially Coupled Series Inductors](image)

The emf that is induced into coil 1 by the effect of the mutual inductance of coil 2 is in opposition to the self-induced emf in coil 1 as now the same current passes through each coil in opposite directions. To take account of this cancelling effect a minus sign is used with \(M\) when the magnetic field of the two coils are differentially connected giving us the final equation for calculating the total inductance of a circuit when the inductors are differentially connected as:

\[
L_{\text{total}} = L_1 + L_2 - 2M
\]

Then the final equation for inductively coupled inductors in series is given as:

\[
L_T = L_1 + L_2 \pm 2M
\]
4.6 Inductors in Parallel

Inductors are said to be connected together in “Parallel” when both of their terminals are respectively connected to each terminal of the other inductor or inductors. The voltage drop across all of the inductors in parallel will be the same. Then, Inductors in Parallel have a Common Voltage across them and in our example below the voltage across the inductors is given as:

\[ V_{L1} = V_{L2} = V_{L3} = V_{AB} \ldots \text{etc} \]

In the following circuit the inductors \( L_1 \), \( L_2 \) and \( L_3 \) are all connected together in parallel between the two points A and B.

4.6.1 Inductors in Parallel Circuit

In the previous series inductors tutorial, we saw that the total inductance, \( L_T \) of the circuit was equal to the sum of all the individual inductors added together. For inductors in parallel the equivalent circuit inductance \( L_T \) is calculated differently. The sum of the individual currents flowing through each inductor can be found using Kirchoff’s Current Law (KCL) where, \( I_T = I_1 + I_2 + I_3 \) and we know from the previous tutorials on inductance that the self-induced emf across an inductor is given as: \( V = L \frac{di}{dt} \). Then by taking the values of the individual currents flowing through each inductor in our circuit above, and substituting the current \( i \) for \( i_1 + i_2 + i_3 \), the voltage across the parallel combination is given as:

\[ V_{AB} = L_T \frac{d}{dt} \left( i_1 + i_2 + i_3 \right) = L_T \left( \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} \right) \]

By substituting \( di/dt \) in the above equation with \( v/L \) gives:

\[ V_{AB} = L_T \left( \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3} \right) \]
4.6.2 Parallel Inductor Equation

\[ \frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_N} \]

Here, like the calculations for parallel resistors, the reciprocal \( \frac{1}{Ln} \) value of the individual inductances are all added together instead of the inductances themselves.

But again as with series connected inductances, the above equation only holds true when there is “NO” mutual inductance or magnetic coupling between two or more of the inductors, (they are magnetically isolated from each other). Where there is coupling between coils, the total inductance is also affected by the amount of coupling. This method of calculation can be used for calculating any number of individual inductances connected together within a single parallel network. If however, there are only two individual inductors in parallel then a much simpler and quicker formula can be used to find the total inductance value, and this is:

\[ L_T = \frac{L_1 \times L_2}{L_1 + L_2} \]

One important point to remember about inductors in parallel circuits, the total inductance \( L_T \) of any two or more inductors connected together in parallel will always be less than the value of the smallest inductance in the parallel chain.

4.6.3 Mutually Coupled Inductors in Parallel

When inductors are connected together in parallel so that the magnetic field of one links with the other, the effect of Mutual Inductance either increases or decreases the total inductance depending upon the amount of magnetic coupling that exists between the coils. The effect of this mutual inductance depends upon the distance apart of the coils and their orientation to each other. Mutually connected inductors in parallel can be classed as either “aiding” or “opposing” the total inductance with parallel aiding connected coils increasing the total equivalent inductance and parallel opposing coils decreasing the total equivalent inductance compared to coils that have zero mutual inductance. Mutual coupled parallel coils can be shown as either connected in an aiding or opposing configuration by the use of polarity dots or polarity markers as shown below.

4.6.4 Parallel Aiding Inductors

![Fig4.9 Parallel Aiding Inductors](image-url)
The voltage across the two parallel aiding inductors above must be equal since they are in parallel so the two currents, \(i_1\) and \(i_2\) must vary so that the voltage across them stays the same. Then the total inductance, \(L_T\) for two parallel aiding inductors is given as:

\[
L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}
\]

Where: 2M represents the influence of coil \(L_1\) on \(L_2\) and likewise coil \(L_2\) on \(L_1\). If the two inductances are equal and the magnetic coupling is perfect such as in a toroidal circuit, then the equivalent inductance of the two inductors in parallel is \(L\) as \(L_T = L_1 = L_2 = M\). However, if the mutual inductance between them is zero, the equivalent inductance would be \(L \div 2\) the same as for two self-induced inductors in parallel. If one of the two coils was reversed with respect to the other, we would then have two parallel opposing inductors and the mutual inductance, \(M\) that exists between the two coils will have a cancelling effect on each coil instead of an aiding effect as shown below.

### 4.6.5 Parallel Opposing Inductors

![Fig4.10 Parallel Opposing Inductors](image)

Then the total inductance, \(L_T\) for two parallel opposing inductors is given as:

\[
L_T = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}
\]

This time, if the two inductances are equal in value and the magnetic coupling is perfect between them, the equivalent inductance and also the self-induced emf across the inductors will be zero as the two inductors cancel each other out. This is because as the two currents, \(i_1\) and \(i_2\) flow through each inductor in turn the total mutual flux generated between them is zero because the two flux’s produced by each inductor are both equal in magnitude but in opposite directions. Then the two coils effectively become a short circuit to the flow of current in the circuit so the equivalent inductance, \(L_T\) becomes equal to \((L \pm M) \div 2\).
4.6.6 Inductors in Parallel

Calculate the equivalent inductance of the following inductive circuit.

![Inductors in Parallel Diagram]

**Fig 4.11 Inductors in Parallel**

Calculate the first inductor branch \( L_A \), (Inductor \( L_5 \) in parallel with inductors \( L_6 \) and \( L_7 \))

\[
L_A = \frac{L_5 \times (L_6 + L_7)}{L_5 + L_6 + L_7} = \frac{50\text{mH} \times (40\text{mH} + 100\text{mH})}{50\text{mH} + 40\text{mH} + 100\text{mH}} = 36.8\text{mH}
\]

Calculate the second inductor branch \( L_B \), (Inductor \( L_3 \) in parallel with inductors \( L_4 \) and \( L_A \))

\[
L_B = \frac{L_3 \times (L_4 + L_A)}{L_3 + L_4 + L_A} = \frac{30\text{mH} \times (20\text{mH} + 36.8\text{mH})}{30\text{mH} + 20\text{mH} + 36.8\text{mH}} = 19.6\text{mH}
\]

Calculate the equivalent circuit inductance \( L_{EQ} \), (Inductor \( L_1 \) in parallel with inductors \( L_2 \) and \( L_B \))

\[
L_{EQ} = \frac{L_1 \times (L_2 + L_B)}{L_1 + L_2 + L_B} = \frac{20\text{mH} \times (40\text{mH} + 19.6\text{mH})}{20\text{mH} + 40\text{mH} + 19.6\text{mH}} = 15\text{mH}
\]

Then the equivalent inductance for the above circuit was found to be: 15mH.
4.7 Tuned Amplifiers

**Tuned Amplifiers**

Amplifiers which amplify a specific frequency or narrow band of frequencies are called **tuned amplifiers**.

Tuned amplifiers are mostly used for the amplification of high or radio frequencies. It is because radio frequencies are generally single and the tuned circuit permits their selection and efficient amplification. However, such amplifiers are not suitable for the amplification of audio frequencies as they are mixture of frequencies from 20 Hz to 20 kHz and not single. Tuned amplifiers are widely used in radio and television circuits where they are called upon to handle radio frequencies.

Fig. 15.1 shows the circuit of a simple transistor tuned amplifier. Here, instead of load resistor, we have a parallel tuned circuit in the collector. The impedance of this tuned circuit strongly depends upon frequency. It offers a very high impedance at **resonant frequency** and very small impedance at all other frequencies. If the signal has the same frequency as the resonant frequency of **LC** circuit, large amplification will result due to high impedance of **LC** circuit at this frequency. When signals of many frequencies are present at the input of tuned amplifier, it will select and strongly amplify the signals of resonant frequency while rejecting all others. Therefore, such amplifiers are very useful in radio receivers to select the signal from one particular broadcasting station when signals of many other frequencies are present at the receiving aerial.
**Resonant frequency.** The frequency at which parallel resonance occurs (i.e. reactive component of circuit current becomes zero) is called the resonant frequency \( f_r \).

At parallel resonance, we have, \( I_C = I_L \sin \phi_L \)

Now \( I_L = \frac{V}{Z_L} \); \( \sin \phi_L = \frac{X_L}{Z_L} \) and \( I_C = \frac{V}{X_C} \)

\[
\begin{align*}
\frac{V}{X_C} & = \frac{V}{Z_L} \times \frac{X_L}{Z_L} \\
X_L \times X_C & = Z_L^2 \\
\frac{\omega L}{\omega C} & = Z_L^2 = R^2 + X_L^2 \\
\frac{L}{C} & = R^2 + (2 \pi f_r L)^2 \\
(2 \pi f_r L)^2 & = \frac{L}{C} - R^2 \\
2 \pi f_r L & = \sqrt{\frac{L}{C} - R^2} \\
f_r & = \frac{1}{2 \pi L} \sqrt{\frac{L}{C} - R^2}
\end{align*}
\]

Resonant frequency, \( f_r = \frac{1}{2 \pi L} \sqrt{\frac{L}{C} - R^2} \)

If coil resistance \( R \) is small (as is generally the case), then,

\[
f_r = \frac{1}{2 \pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}
\]

The resonant frequency will be in Hz if \( R, L \) and \( C \) are in ohms, henry and farad respectively.

**Note.** If in the problem, the value of \( R \) is given, then eq. (ii) should be used to find \( f_r \). However, if \( R \) is not given, then eq. (iii) may be used to find \( f_r \).

**Characteristics of Parallel Resonant Circuit**

It is now desirable to discuss some important characteristics of parallel resonant circuit.

(i) **Impedance of tuned circuit.** The impedance offered by the parallel \( LC \) circuit is given by the supply voltage divided by the line current i.e., \( V/I \). Since at resonance, line current is minimum, therefore, impedance is maximum at resonant frequency. This fact is shown by the impedance-fre-
quency curve of Fig 15.5. It is clear from impedance-frequency curve that impedance rises to a steep peak at resonant frequency $f_r$. However, the impedance of the circuit decreases rapidly when the frequency is changed above or below the resonant frequency. This characteristic of parallel tuned circuit provides it the selective properties i.e. to select the resonant frequency and reject all others.

Line current, \[ I = I_L \cos \theta_L \]
\[
\frac{V}{Z_r} = \frac{V}{Z_L} \times \frac{R}{Z_L}
\]
\[
\frac{1}{Z_r} = \frac{R}{Z_L} = \frac{C}{L} \quad \text{from eq. (i)}
\]
Circuit impedance, \[ Z_r = \frac{L}{C_R} \]

Thus at parallel resonance, the circuit impedance is equal to \[ L/C \]. It may be noted that $Z_r$ will be in ohms if $R, L$ and $C$ are measured in ohms, henry and farad respectively.

\( \text{(ii) Circuit Current.} \) At parallel resonance, the circuit or line current $I$ is given by the applied voltage divided by the circuit impedance $Z_r$, i.e.,

\[ I = \frac{V}{Z_r} \quad \text{where} \quad Z_r = \frac{L}{C_R} \]

Because $Z_r$ is very high, the line current $I$ will be very small.

\( \text{(iii) Quality factor} \quad Q. \) It is desired that resonance curve of a parallel tuned circuit should be as sharp as possible in order to provide selectivity. The sharp resonance curve means that impedance falls rapidly as the frequency is varied from the resonant frequency. The smaller the resistance of coil, the more sharp is the resonance curve. This is due to the fact that a small resistance consumes less power and draws a relatively small line current. The ratio of inductive reactance and resistance of the coil at resonance, therefore, becomes a measure of the quality of the tuned circuit. This is called quality factor and may be defined as under:

The ratio of inductive reactance of the coil at resonance to its resistance is known as quality factor $Q$ i.e.,

\[ Q = \frac{X_L}{R} = \frac{2\pi f_r L}{R} \]

The quality factor $Q$ of a parallel tuned circuit is very important because the sharpness of resonance curve and hence selectivity of the circuit depends upon it. The higher the value of $Q$, the more selective is the tuned circuit. Fig. 15.6 shows the effect of resistance $R$ of the coil on the sharpness of

Two things are worth noting. First, $Z_r = L/C$ is a pure resistance because there is no frequency term present. Secondly, the value of $Z_r$ is very high because the ratio $L/C$ is very large at parallel resonance.

Strictly speaking, the $Q$ of a tank circuit is defined as the ratio of the energy stored in the circuit to the energy lost in the circuit i.e.,

\[ Q = \frac{\text{Energy stored}}{\text{Energy lost}} = \frac{\text{Reactive Power}}{\text{Resistive Power}} = \frac{I_L^2 X_L}{I_L^2 R} \quad \text{or} \quad Q = \frac{X_L}{R} \]
Problem 2  A parallel resonant circuit has a capacitor of 100 pF in one branch and inductance of 100 µH plus a resistance of 10 Ω in parallel branch. If the supply voltage is 10 V, calculate (i) resonant frequency (ii) impedance of the circuit and line current at resonance.

Solution.

\( R = 10 \, \Omega \), \( L = 100 \times 10^{-6} \, \text{H} \), \( C = 100 \times 10^{-12} \, \text{F} \)

(i) Resonant frequency of the circuit is

\[
f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}
\]

\[
= \frac{1}{2\pi} \sqrt{\frac{10^{12}}{100 \times 10^{-6} \times 100} - \frac{10^2}{(100 \times 10^{-6})^2}} \, \text{Hz}
\]

\[
= 1592.28 \times 10^3 \, \text{Hz} = 1592.28 \, \text{kHz}
\]

(ii) Impedance of the circuit at resonance is

\[
Z_r = \frac{L}{C \cdot R} = \frac{L}{C} \times \frac{1}{R} = \frac{100 \times 10^{-6}}{100 \times 10^{-12}} \times \frac{1}{R}
\]
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\[
= 10^6 \times \frac{1}{R} = 10^6 \times \frac{1}{10} = 10^5 \Omega = 0.1 \text{ M\Omega}
\]

Note that the circuit impedance \( Z_r \) is very high at resonance. It is because the ratio \( L/C \) is very large at resonance.

Line current at resonance is

\[
I = \frac{V}{Z_r} = \frac{10 \text{ V}}{10^5 \Omega} = 100 \mu\text{A}
\]

**Problem 3** The dynamic impedance of a parallel resonant circuit is 500 k\( \Omega \). The circuit consists of a 250 pF capacitor in parallel with a coil of resistance 10\( \Omega \). Calculate (i) the coil inductance (ii) the resonant frequency and (iii) \( Q \)-factor of the circuit.

**Solution.**

(i) Dynamic impedance, \( Z_r = \frac{L}{CR} \)

\[
\therefore \text{ Inductance of coil, } L = Z_r CR = (500 \times 10^3) \times (250 \times 10^{-12}) \times 10
\]

\[
= 1.25 \times 10^{-3} \text{ H} = 1.25 \text{ mH}
\]

(ii) Resonant frequency,

\[
f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}
\]

\[
= \frac{1}{2\pi} \sqrt{\frac{10^3}{1.25 \times 10^{-3} \times 250} - \frac{10^2}{(1.25 \times 10^{-3})^2}}
\]

\[
= 284.7 \times 10^3 \text{ Hz} = 284.7 \text{ kHz}
\]

(iii) \( Q \)-factor of the circuit

\[
\frac{2\pi f_r L}{R} = \frac{2\pi \times (284.7 \times 10^3) \times (1.25 \times 10^{-3})}{10} = 223.6
\]

**Advantages of Tuned Amplifiers**

In high frequency applications, it is generally required to amplify a single frequency, rejecting all other frequencies present. For such purposes, tuned amplifiers are used. These amplifiers use tuned parallel circuit as the collector load and offer the following advantages:

(i) **Small power loss**. A tuned parallel circuit employs reactive components \( L \) and \( C \). Consequently, the power loss in such a circuit is quite low. On the other hand, if a resistive load is used in the collector circuit, there will be considerable loss of power. Therefore, tuned amplifiers are highly efficient.

(ii) **High selectivity**. A tuned circuit has the property of selectivity i.e. it can select the desired frequency for amplification out of a large number of frequencies simultaneously impressed upon it. For instance, if a mixture of frequencies including \( f_r \) is fed to the input of a tuned amplifier, then maximum amplification occurs for \( f_r \). For all other frequencies, the tuned circuit offers very low impedance and hence these are amplified to a little extent and may be thought as rejected by the circuit. On the other hand, if we use resistive load in the collector, all the frequencies will be amplified equally well i.e. the circuit will not have the ability to select the desired frequency.

(iii) **Smaller collector supply voltage**. Because of little resistance in the parallel tuned circuit, it requires small collector supply voltage \( V_{CC} \). On the other hand, if a high load resistance is used in the collector for amplifying even one frequency, it would mean large voltage drop across it due to zero signal collector current. Consequently, a higher collector supply will be needed.
Frequency Response of Tuned Amplifier

The voltage gain of an amplifier depends upon $\beta$, input impedance and effective collector load. In a tuned amplifier, tuned circuit is used in the collector. Therefore, voltage gain of such an amplifier is given by:

$$\text{Voltage gain} = \frac{\beta Z_C}{Z_{in}}$$

where

$Z_C$ = effective collector load
$Z_{in}$ = input impedance of the amplifier

The value of $Z_C$ and hence gain strongly depends upon frequency in the tuned amplifier. As $Z_C$ is maximum at resonant frequency, therefore, voltage gain will be maximum at this frequency. The value of $Z_C$ and gain decrease as the frequency is varied above and below the resonant frequency. Fig. 15.7 shows the frequency response of a tuned amplifier. It is clear that voltage gain is maximum at resonant frequency and falls off as the frequency is varied in either direction from resonance.

**Bandwidth.** The range of frequencies at which the voltage gain of the tuned amplifier falls to 70.7% of the maximum gain is called its *bandwidth*. Referring to Fig.

the bandwidth of tuned amplifier is $f_1 - f_2$. The amplifier will amplify nicely any signal in this frequency range. The bandwidth of tuned amplifier depends upon the value of $Q$ of $LC$ circuit *i.e.* upon the sharpness of the frequency response. The greater the value of $Q$ of tuned circuit, the lesser is the bandwidth of the amplifier and *vice-versa*. In practice, the value of $Q$ of $LC$ circuit is made such so as to permit the amplification of desired narrow band of high frequencies.

Relation between $Q$ and Bandwidth

The quality factor $Q$ of a tuned amplifier is equal to the ratio of resonant frequency ($f_r$) to bandwidth ($BW$) *i.e.,*

$$Q = \frac{f_r}{BW}$$

The $Q$ of an amplifier is determined by the circuit component values. It may be noted here that $Q$ of a tuned amplifier is generally greater than 10. When this condition is met, the resonant frequency at parallel resonance is approximately given by:

$$f_r = \frac{1}{2\pi \sqrt{LC}}$$
**Single Tuned Amplifier**

A single tuned amplifier consists of a transistor amplifier containing a parallel tuned circuit as the collector load. The values of capacitance and inductance of the tuned circuit are so selected that its resonant frequency is equal to the frequency to be amplified. The output from a single tuned amplifier can be obtained either by a coupling capacitor $C_C$.

![Amplifier Circuit Diagram]

**Operation.** The high frequency signal to be amplified is given to the input of the amplifier. The resonant frequency of the parallel tuned circuit is made equal to the frequency of the signal by changing the value of $C$. Under such conditions, the tuned circuit will offer very high impedance to the signal frequency. Hence a large output appears across the tuned circuit. In case the input signal is complex containing many frequencies, only that frequency which corresponds to the resonant frequency of the tuned circuit will be amplified. All other frequencies will be rejected by the tuned circuit. In this way, a tuned amplifier selects and amplifies the desired frequency.

**Note.** The fundamental difference between AF and tuned (RF) amplifiers is the bandwidth they are expected to amplify. The AF amplifiers amplify a major portion of AF spectrum (20 Hz to 20 kHz) equally well throughout. The tuned amplifiers amplify a relatively narrow portion of RF spectrum, rejecting all other frequencies.
4.8 Double Tuned Amplifier

**Double Tuned Amplifier**

Fig. shows the circuit of a double tuned amplifier. It consists of a transistor amplifier containing two tuned circuits: one \((L_1C_1)\) in the collector and the other \((L_2C_2)\) in the output as shown. The high frequency signal to be amplified is applied to the input terminals of the amplifier. The resonant frequency of tuned circuit \(L_1C_1\) is made equal to the signal frequency. Under such conditions, the

At frequencies below \(f_r\), \(X_C > X_L\) or \(I_L > I_C\). Therefore, the circuit will be inductive.
tuned circuit offers very high impedance to the signal frequency. Consequently, large output appears across the tuned circuit $L_1C_1$. The output from this tuned circuit is transferred to the second tuned circuit $L_2C_2$ through mutual induction. Double tuned circuits are extensively used for coupling the various circuits of radio and television receivers.

**Frequency response.** The frequency response of a double tuned circuit depends upon the degree of coupling i.e. upon the amount of mutual inductance between the two tuned circuits. When coil $L_2$ is coupled to coil $L_1$, a portion of load resistance is coupled into the primary tank circuit $L_1C_1$ and affects the primary circuit in exactly the same manner as though a resistor had been added in series with the primary coil $L_1$.

When the coils are spaced apart, all the primary coil $L_1$ flux will not link the secondary coil $L_2$. The coils are said to have *loose coupling*. Under such conditions, the resistance reflected from the load (i.e. secondary circuit) is small. The resonance curve will be sharp and the circuit $Q$ is high as shown in Fig. When the primary and secondary coils are very close together, they are said to have *tight coupling*. Under such conditions, the reflected resistance will be large and the circuit $Q$ is lower. Two positions of gain maxima, one above and the other below the resonant frequency, are obtained.

**Bandwidth of Double-Tuned Circuit**

If you refer to the frequency response of double-tuned circuit shown in Fig. it is clear that bandwidth increases with the degree of coupling. Obviously, the determining factor in a double-tuned circuit is not $Q$ but the coupling. For a given frequency, the tighter the coupling, the greater is the bandwidth.

$$BW_{dt} = kf_r$$

The subscript $dt$ is used to indicate double-tuned circuit. Here $k$ is coefficient of coupling.
Practical Application of Double Tuned Amplifier

Double tuned amplifiers are used for amplifying radio-frequency (RF) signals. One such application is in the radio receiver as shown in Fig. 15.16. This is the IF stage using double tuned resonant circuits. Each resonant circuit is tuned to 455 kHz. The critical coupling occurs when the coefficient of coupling is

\[ k_{critical} = \frac{1}{\sqrt{Q_1 Q_2}} \]

where

- \( Q_1 \) = quality factor of primary resonant circuit \((L_1, C_1)\)
- \( Q_2 \) = quality factor of secondary resonant circuit \((L_2, C_2)\)

When two resonant circuits are critically coupled, the frequency response becomes flat over a considerable range of frequencies as shown in Fig. 15.16. In other words, the double tuned circuit has better frequency response as compared to that of a single tuned circuit. The use of double tuned circuit offers the following advantages:

(i) Bandwidth is increased.
(ii) Sensitivity (i.e. ability to receive weak signals) is increased.
(iii) Selectivity (i.e. ability to discriminate against signals in adjacent bands) is increased.